EARIN

Laboratory report

EXERCISE 7: Bayes models

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Table of contents

[1. Introduction 1](#_Toc105953851)

[2. Theoretical background 1](#_Toc105953852)

[3. Implementation 3](#_Toc105953853)

[4. Results 5](#_Toc105953854)

# Introduction

For the purpose of the laboratory basic inference in Bayesian networks using the MCMC algorithm with Gibbs sampling was performed. Program was written to return the probability distribution of query variables and allows to set the number of steps performed by MCMC algorithm. Estimated data distribution depending on number of steps was illustrated.

# Theoretical background

A Bayesian network is a directed acyclic graph for which:

* the vertices/nodes correspond to discrete random variables/discrete events,
* the edges represent directly occurring dependencies between these variables,
* for each variable there is a probability distribution defined – this distribution is conditional if there are edges lead into the variable.

Within this exercise 2 nodes are defined – test and cancer. Task explicitly states values of probability of having cancer and conditional probabilities of having cancer depending on the result of the test. Values of probabilities of positive and negative tests can be calculated using following property:

|  |  |
| --- | --- |
|  | (1) |

Using equation 1, one may assume that P(Test = T) is equal to x. Therefore, P(Test = F) is equal to 1 – x. Plugging that into the equation the only unknown variable remains x, which as a result can be simply calculated. Knowing values of probabilities of test, one may apply following equation to find conditional probabilities of test depending on cancer:

|  |  |
| --- | --- |
| = | (2) |

Further, one may consider Markov Chain Monte Carlo (MCMC) algorithms. Markov Chain is a model describing a sequence of possible events. Probability of each event within the chain depends only on the state of the previous event. Incorporation of a Markov Chain into Monte Carlo methods results in creation of a random walk. Frequency of visiting particular states during the walk tend to converge to the value of its probability, given the sufficiently high number of steps were simulated.

Gibbs sampling, which is considered to be a special case of the Metropolis–Hastings algorithm, is commonly used for statistical inference and involves choosing a new sample for each dimension separately from the others. In the beginning, we choose randomly values of initial states of all parameters, in our case test and cancer. Then “n” times, where “n” should be sufficiently big number, we determine states of all of the parameters based on the previous state and conditional probabilities. First parameter is chosen entirely on the basis of the values obtained in previous iteration. However, when new sample is chosen for the first of the parameters in a given iteration, it is used later to determine other parameters, rather than the value from the previous state. Similarly, when more than two variables would be considered, the latest values of all parameters would be considered within the conditional probability for j-th variable as presented below:

|  |  |
| --- | --- |
|  | (3) |

Where d denotes total number of variables and i is an iteration number. In the end, one may assess the probability of occurrence of all possible states by summing number of occurrences of a given state and dividing it by the number of iterations. Additionally, some number of initial samples may be discarded as they are considered to be a part of a so called burn-in period

# Implementation

For the purpose of this exercise following set of probabilities was used.

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**Figure 1. Probabilities**

As a result following confusion matrix was expected:

**Table 1. Confusion matrix - probabilities**

|  |  |  |
| --- | --- | --- |
|  | **Test = T** | **Test = F** |
| **Cancer = T** | 0.045 | 0.005 |
| **Cancer = F** | 0.19 | 0.76 |

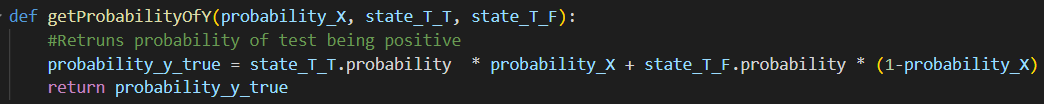
Our algorithm was implemented within the main function and was composed of two main components – selection of initial values and random walk with “k” iterations selected by the user.

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**Figure 2. main() function**

In order to draw initial state values we had to calculate value of probability of test being positive.



**Figure 3. Calculation of probability of test being positive**

Having determined that, we moved to the selection of initial state. We have performed it using getState() and drawRandomState() functions.

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**Figure 4. State selection**

Subsequently, we performed random walk using changeState() and randomWalk() functions.

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**Figure 5. Random walk**

Finally, after performing “k” steps of random walk, we printed results to the console.

# Results

We have performed exemplary tests for 1000 and 100000 steps. Results can be found on figures 6-7, below.

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**Figure 6. Exemplary results for 1000 steps**

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**Figure 7. Exemplary results for 100000 steps**